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Declaration: By submitting this assignment I confirm that it is all my own work and that all references and quotations from primary and secondary sources have been fully identified and properly acknowledged.	
FEEDBACK WILL BE PROVIDED ELECTRONICALLY, TYPICALLY 4 WEEKS AFTER THE SUBMISSION DEADLINE.	

Date: 28.02.2015 **Wolfgang Krispler**

Title: Experiment 2 – Investigation of the Crab Nebula

Abstract

By comparing 2 images of the Crab nebula (M1) taken 27 years apart (1973 and 2000) the angular velocity of the expansion for 10 knots in the nebula's filaments has been calculated. By measurements in the emission spectrum the maximum linear velocity of the expansion along the line of sight was estimated.

The measurements lead to the conclusion that the Crab nebula is approximately 6625 light years away (the accepted distance is 6500 light years according to Universe Tenth edition page 6) and the date of the expansion is 1268+/-130.

The biggest uncertainty in the result is the unknown 3D shape of the nebula's shell. Assuming that the nebula has been expanding at a constant rate the angular velocity calculations date the supernova more than 200 years after the accepted date of the supernova in 1054.

Introduction

In 1054 a supernova with an apparent magnitude of -4, lasting for about 2 years, was observed. The remnant which consists of debris ejected during the explosion is known as crab nebula (M1).

The angular velocity of 10 identifiable knots in the filaments at the nebula's periphery was calculated by measuring the angular distance of the knots using images of the Crab nebula from 1973 and 2000.

With the angular velocity and the measured angular distance to the Pulsar in the center of the crab nebula the age of the explosion was calculated.

By calculating the linear velocity from measurements of the nebula's spectrum and the usage of the proper motion distance formula the distance to the Crab nebula was estimated.

Part 1 - Finding the Nebula's Age

Procedure/Method

1. Photoshop Elements 13 was used to edit the photos and bring out more contrast at the periphery of the nebula to facilitate the identification of the filament knots
2. The edited images (b/w 612x691 pixels) from 1973 and 2000 were printed on a DIN A4 paper for the measurements
3. A 225mm triangle ruler was used to make the measurements. It is marked down to 1mm and by careful estimation it was possible to make measurements of 0.1mm
4. Excel 2013 was used to process the data and provide the plotting.

Measurements

All measurements were done on the edited versions of the provided nebula images from the years 1973 and 2000.

The image of 1973 was used to check the reliability of the measurements. The distance of the dark center of knot 1 to the closest limb of the Pulsar (PSR B0531+21) was measured 5 times on 5 different days.

	M1	M2	M3	M4	M5
Distance to Pulsar 1973 [mm]	84,6	84,8	84,9	84,7	84,5

The standard deviation of 0.16 mm allows the conclusion that the measurements are good to better than 0.2mm. This includes the uncertainty with reading the ruler and the additional component due to the positioning of the ruler itself.

Image1 Edited image of the Crab Nebula from 1973 identifying reference stars A and B, pulsar P, 10 knots (1..10) and 5 reference stars (S1..S5)

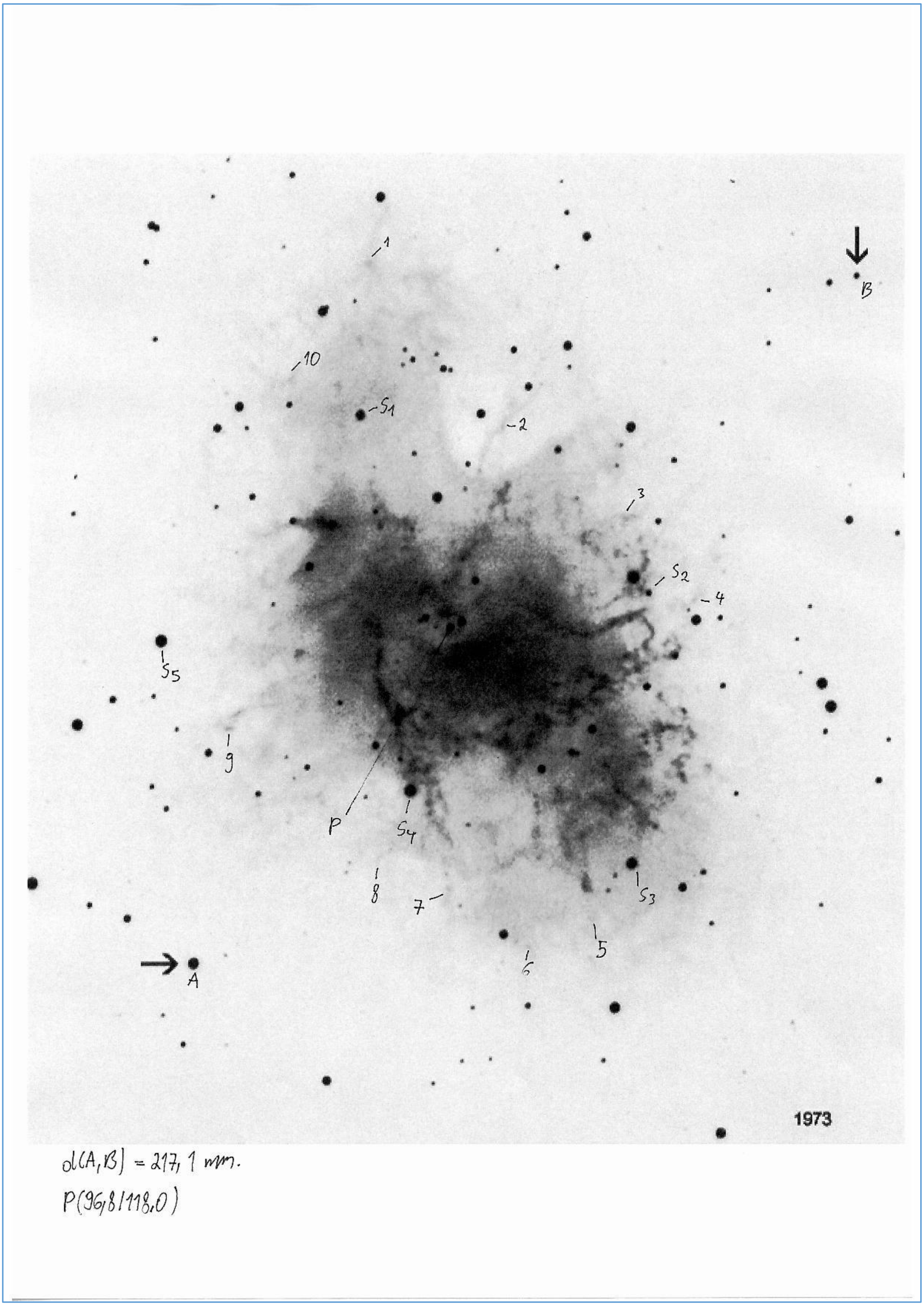
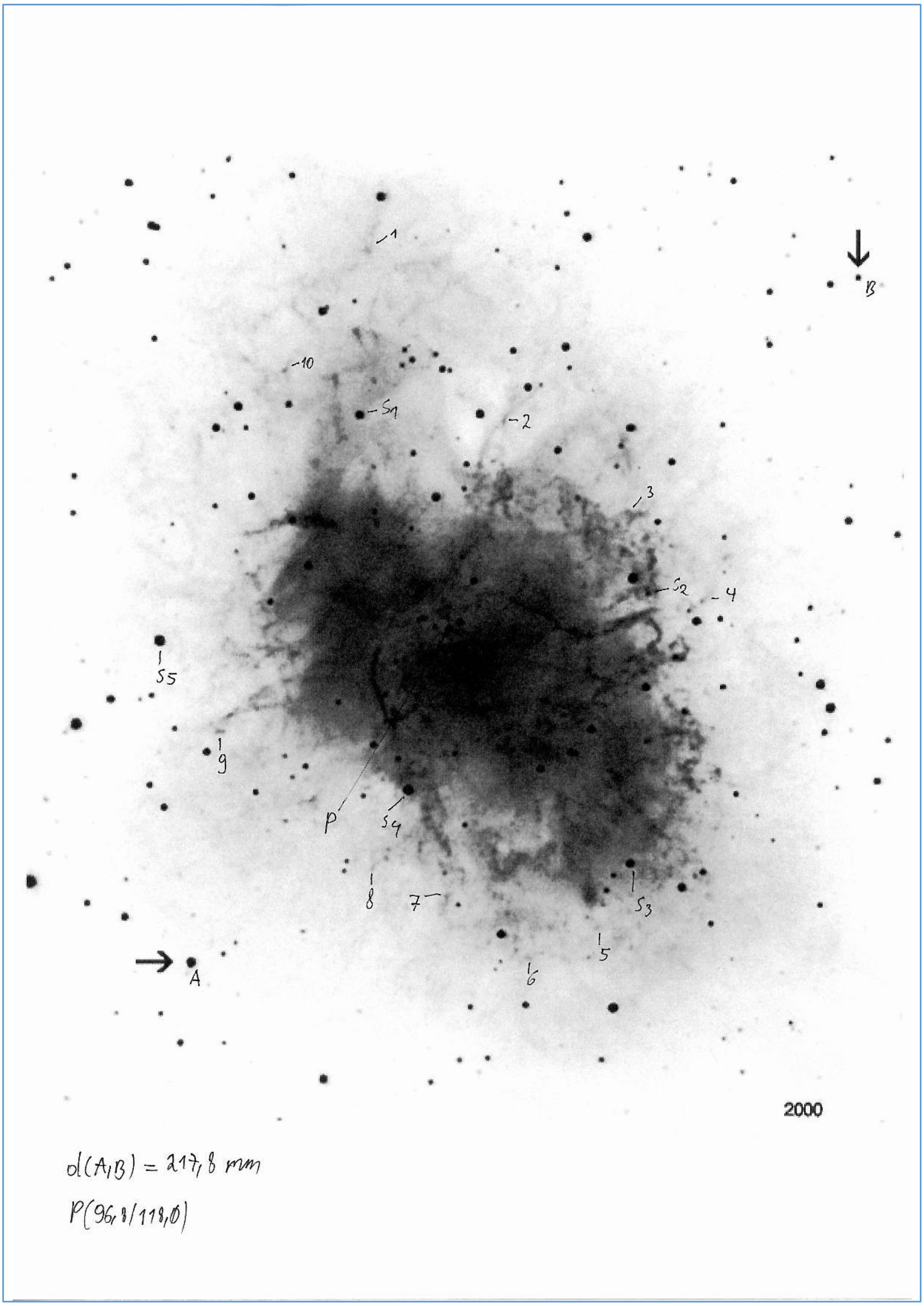


Image2 Edited image of the Crab Nebula from 2000 identifying reference stars A and B, pulsar P, 10 knots and 5 reference stars



In order to establish a photographic scale the distances between the marked stars A and B were measured 5 times on each of the 2 images and the adopted average value transcribed to table 1. The error of +/-0,2mm was calculated with the standard deviation function of Excel.

The angular distance between the two stars was given with 385 seconds of arc with zero error leading to a scale of 1, 77+/-0,002 arc seconds per mm on both images.

The error was calculated with the formula for error in quotient:

$$\frac{\Delta Q}{Q} = \sqrt{(\frac{\Delta x}{x})^2 + (\frac{\Delta y}{y})^2}$$

$$Q = \frac{385}{217{,}1} = 1{,}77$$
$$\Delta x = 0{,}2$$
$$\Delta y = 0$$

$$\Delta Q = 0{,}002\ arcseconds\ per\ mm$$

Table 1 Photographic scale in arc seconds per mm derived from the measured distances between stars A & B

Date	Distance between marked Stars (mm)	Error [mm]	Photographic Scale (arcseconds/mm)	Error [arcseconds/mm]
1973	217,1	+/- 0,2	1,77	0,002
2000	217,8	+/- 0,2	1,77	0,002

The position of the Pulsar P (PSR B0531+21) the southernmost of the two close central stars, was measured in millimeters from the lower left corner on each image. The position of the pulsar is identical on both pictures with P (96,6mm/118mm).

On both images 10 identifiable knots on the periphery of the nebula have been located and marked (1..10) to increase the measure distances and consequently decreasing the effect of measurement error.

Next the distance from each of the knots to the pulsar was measured 5 times within 1 week with a millimeter ruler to tenths of a millimeter. The measurement was done from the dark center of the knot (as the limbs were hard to identify) to the closest limb of the pulsar.

As a control 5 stars in the outskirts of the nebula have been marked and measured as well 5 times within 1 week.

The adopted average values can be found in the Table 2 beneath in the section calculations.

Calculations and Error Estimates

The linear measurements in millimeters were converted to angular distances in seconds of arcs using the 2 plate's scales.

For example, the average measured distance of knot 1 on the image of 1973 to the pulsar was 84,6mm. Using the plate scale (1mm=1,77arcseconds) this leads to an angular distance of 149, 7 seconds of arc for knot 1.

The error was calculated with the formula for fractional error in product:

$$\frac{\Delta Q}{Q} = \sqrt{(\frac{\Delta x}{x})^2 + (\frac{\Delta y}{y})^2}$$

$$Q = 84{,}6 * 1{,}77 = 149{,}7$$
$$\Delta x = 0{,}2$$
$$\Delta y = 0{,}\ 002$$

$$\Delta Q = 0{,}39\ arcseconds$$

The error in arc seconds for the other knots was calculated using the same formula.

The angular velocity of any knot is given by the expression

$$\mu = \frac{\Delta x}{\Delta t}$$

where $\boldsymbol{\mu}$ is the angular velocity or proper motion, Δx is the angular change in the position of the knot and Δt is the time interval between the plates, in our example 27 years. The angular velocity of each knot was calculated in arc seconds per year.

E.g. for knot 1 the calculation is done as follows:

Δx..6,02 arcseconds – The knot moved from 1973 to 2000 6,02 arc seconds
Δt..27 years

$$\mu = \frac{6{,}02}{27} = 0{,}22\ arcseconds\ per\ year$$

The angular velocity of knot 1 is 0,22 seconds of arc per year.

The same calculation is done for all knots, the results including the errors (using the formulas described above) are displayed in Table 2.

The total time T since the explosion can now be calculated by the relation

$$T = \frac{x}{\mu}$$

where x is the distance of the knot to the pulsar in 2000 in arc seconds and $\boldsymbol{\mu}$ is the angular velocity of the knot.

E.g. for knot 1 the calculation is done as follows:

x..155,8 arc seconds
$\boldsymbol{\mu}$..0,22 arc seconds per year

$T = \frac{x}{\mu} = \frac{155{,}8}{0{,}22}$ =708 years

$$\frac{\Delta Q}{Q} = \sqrt{(\frac{\Delta x}{x})^2 + (\frac{\Delta y}{y})^2}$$

$$Q = \frac{155{,}8}{0{,}22} = 708$$
$\Delta x = 0{,}4$
$\Delta y = 0{,}02$

$\Delta Q = 66$ years

Table 2 Measurement and calculation values of 10 knots of the crab nebula and 5 control stars

ID	Distance to Pulsar 1973	Error (+/-)	Distance to Pulsar 1973	Error (+/-)	Distance to Pulsar 2000	Error (+/-)	Distance to Pulsar 2000	Error (+/-)	Delta	Error for Delta (+/-)	Angular velocity	Error for angular velocity (+/-)	Convergence age	Error for conv. Age [+/-)
	[mm]	[mm]	[arcseconds]	[arcseconds]	[mm]	[mm]	[arcseconds]	[arcseconds]	[arcseconds]	[arcseconds]	[arcseconds/year]	[arcseconds/year]	[years]	[years]
1	84,6	0,2	149,7	0,39	88,0	0,2	155,76	0,40	6,02	0,56	0,22	0,02	708	66
2	46,6	0,2	82,5	0,37	48,4	0,2	85,67	0,37	3,19	0,52	0,12	0,02	714	126
3	46,4	0,2	82,1	0,37	48,5	0,2	85,85	0,37	3,72	0,52	0,14	0,02	613	92
4	55,6	0,2	98,4	0,37	57,3	0,2	101,42	0,37	3,01	0,53	0,11	0,02	922	172
5	72,9	0,2	129,0	0,38	75,5	0,2	133,64	0,38	4,60	0,54	0,17	0,02	786	96
6	73,8	0,2	130,6	0,38	76,8	0,2	135,94	0,39	5,31	0,54	0,20	0,02	680	72
7	58,7	0,2	103,9	0,37	60,5	0,2	107,09	0,37	3,19	0,53	0,12	0,02	892	157
8	53,8	0,2	95,2	0,37	56,8	0,2	100,54	0,37	5,31	0,52	0,20	0,02	503	53
9	55	0,2	97,4	0,37	56,8	0,2	100,54	0,37	3,19	0,53	0,12	0,02	838	147
10	67,1	0,2	118,8	0,38	69,4	0,2	122,84	0,38	4,07	0,54	0,15	0,02	819	113
S1	51	0,2	90,3	0,37	51,1	0,2	90,45	0,37	0,18	0,52	0,01	0,03	n/a	n/a
S2	44,8	0,2	79,3	0,37	44,9	0,2	79,47	0,37	0,18	0,52	0,01	0,03	n/a	n/a
S3	65,8	0,2	116,5	0,38	66,0	0,2	116,82	0,38	0,35	0,53	0,01	0,02	n/a	n/a
S4	36,2	0,2	64,1	0,36	36,4	0,2	64,43	0,36	0,35	0,51	0,01	0,02	n/a	n/a
S5	64	0,2	113,3	0,38	64,2	0,2	113,63	0,38	0,35	0,53	0,01	0,02	n/a	n/a

The average value for T of all measured knots was calculated with the average function in Excel and results in 747 years with a standard deviation of 130 years. According to this calculations the date of the expansion would be 1268+/-130.

Result and Discussion

All calculations above assume a constant angular velocity. The angular velocities of the knots differed depending on where the knot lies as the one with the greatest angular velocities are those with no velocity towards or away from earth, the knots on the outer age of the nebula.

The average predicted age for the nebula was 747 year with a standard deviation of 130 years. These calculations predicted a convergence of the nebula to the pulsar in the year 1268 more than 200 years later than 1054. Conclusions which may be derived from this result can be found beneath in the conclusion paragraph.

The standard deviation of 130 years is small taking into consideration that e.g. a measurement error of only 0.5mm for knot 1 would lead to a shift of 109! years of the convergence age compared to the value in table 2.

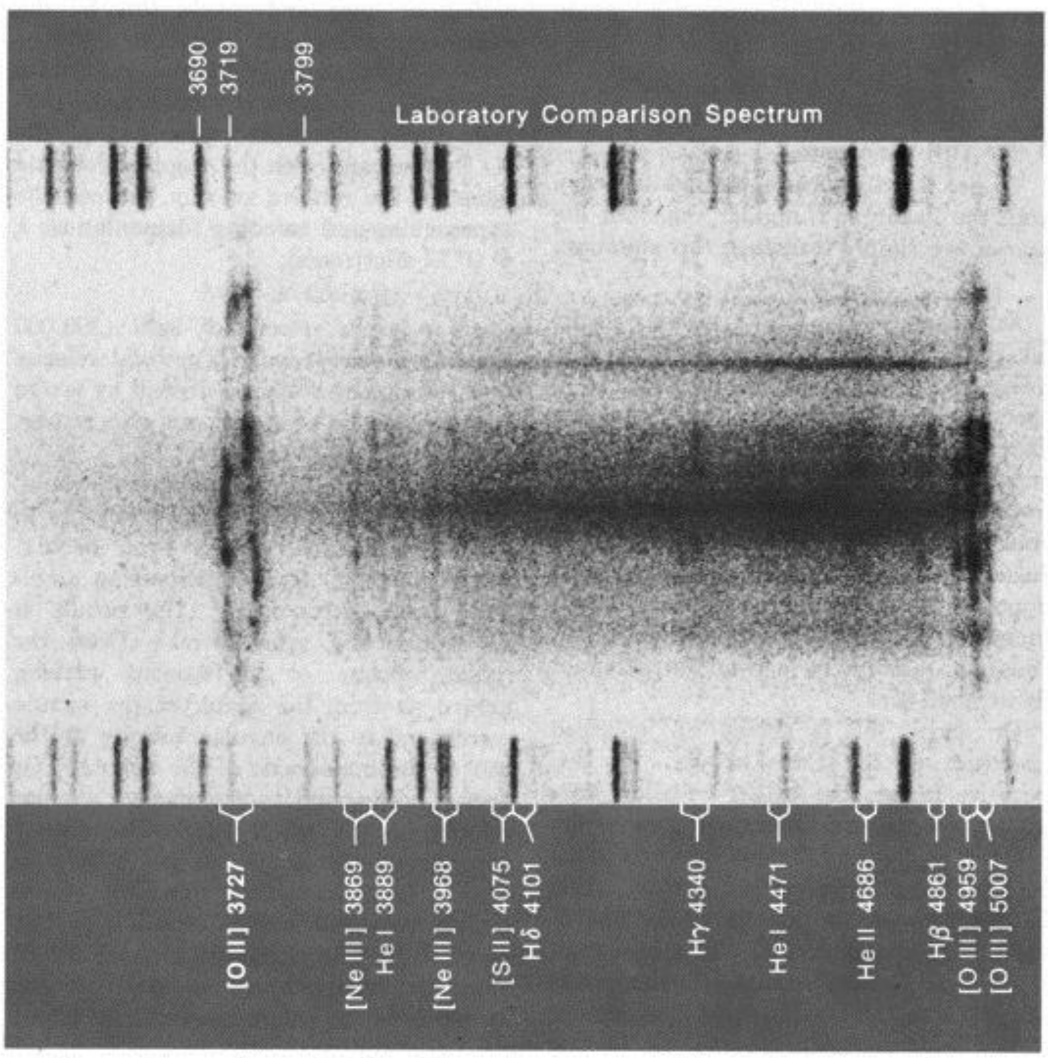
Part 2 - Finding Distance to Nebula by Expansion Parallax

Procedure/Method

1. The image (b/w 747x894 pixels) of the spectrum was printed on a DIN A4 paper for the measurements
2. A 225mm triangle ruler was used to make the measurements. It is marked down to 1mm and by careful estimation it was possible to make measurements of 0.1mm
3. Excel 2013 was used to process the data and provide the plotting.

Measurements

Image 3 – Spectrum of the Crab Nebula (including laboratory comparison spectrum)



The spectrum of the Crab nebula, obtained at Lick Observatory by N. U. Mayall with the Crossley reflector. The spectrograph slit was aligned with the major axis of the nebula (here vertical), to record velocity differences along that axis. These are best shown by the necklace shape of the 3727-angstrom oxygen line. A laboratory spectrum of palladium, tin, and lead flanks that of the Crab to give a wavelength scale; nebular lines are identified at bottom.

First a dispersion scale was established by measuring the center to center distance between the 369.0 nm and 379.9 palladium line from the laboratory comparison spectrum of the element palladium.

An average distance of 16,2+/-0,2 mm was measured which results in a dispersion scale of 0,67+/-0,01 nm per mm.

The error of 0,01 nm per mm was again calculated by the formula for the fractional error in quotient

$$\frac{\Delta Q}{Q} = \sqrt{(\frac{\Delta x}{x})^2 + (\frac{\Delta y}{y})^2}$$

$$Q = \frac{10,9}{16,2} = 0,67$$
$$\Delta x = 0$$
$$\Delta y = 0,2$$

$$\Delta Q = 0,01 nm$$

Next the maximum shift between the red-shifted and blue-shifted branches of the [OIII] line was measured with 5,4+/-0,2mm. Using the above calculated dispersion scale (0,67+/-0,01nm/mm) the maximum shift is 3,6+/-0,13 nm.

Calculations and Error Estimates

Using the Doppler formula

$$\frac{\Delta\lambda}{\lambda_0} = \frac{v}{c}$$

$\lambda_0 = 373.7nm$; v..relative velocity between approaching and receding filaments, c...300.000 km/s

We calculate the relative velocity between the approaching and receding filaments.

$$v = \frac{\Delta\lambda}{\lambda_0} = \frac{3,6nm}{373.7\text{nm}} * 300.000\text{km}/s$$

v=2890km/s

The error of 0,13 nm in this calculation was neglected.

By dividing by 2 we obtain a linear velocity of the knots moving outwards from the pulsar of 1445 km per second.

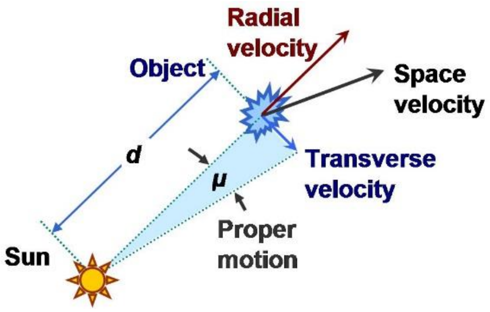
In order to calculate the distance to the Crab nebula we use the proper motion formula.

The transverse motion across the sky is called proper Motion (μ) measured in arc seconds per year.
The unit of measure typically used to describe stellar distance is the parsec (pc), and is defined as the distance at which one astronomical unit (AU) subtends one arc second in the sky. So, an angle of μ arc sec at a distance of r pc corresponds to a separation of rμ AU; proper motion of μ arc sec per year of a star at a distance of r pc corresponds therefore to a velocity of rμ AU per year. 1 AU = 1.5 × 1011 m approximately 1 year = 3 × 107 s approximately Therefore 1 AU per year = 5 × 103 m · s −1 approximately.

$$d = \frac{v}{4,74 * \mu}$$

d..distance in parsec
v..linear velocity (km/s)
μ..proper motion (seconds of arc per year)

Image 4 – Proper motion of an object



$$d = \frac{1445}{4{,}74 * \mu}$$

The maximum angular velocity in table 2 is 0,22 arc seconds per year which would give a distance of 1386 pc (4517 light years.
Taking the minimum angular velocity of 0,11 arc seconds per year the distance of the nebula would be 2771 pc (9035 light years).
By using the average angular velocity of 0,15 arc seconds per year the distance of the nebula is 2032pc (6625 light years).

According to Chinese records the apparent magnitude of the supernova reached m_v=-4

We use the modulus distance formula and the average distance to calculate the absolute magnitude M_v.

$$\mathrm{mv} - \mathrm{Mv} = 5 * logd - 5$$

$$-4 - Mv = 5 * log2771 - 5$$
M_v=-15.54

Based on a distance of 2032pc and an apparent magnitude of -4 the supernova had an absolute magnitude of -15.54 which is brighter than the most luminous star of the Hipparcos star catalogue Rigel (HIP 24236) with an absolute magnitude of -6.69.

Result and Discussion

When calculating the angular velocity we calculate the angular velocity of each knot. When calculating the linear velocity based on the maximum distance between the red-shifted and blue-shifted branches of the [OIII] lines we only know the maximum linear velocity along the line of sight! We do not know the shape of the nebula. Therefore we only can take the average angular velocity in order to try to calculate the distance to the middle of the nebula. If we would know beside the angular velocity of each knot the linear velocity of each knot we could make a statement about the shape of the nebula.
The best estimate of the distance to the nebula is 6625 light years although for sure is only that is lies between 4517 and 9035 light years.

Conclusion

The expansion rate of the Crab nebula, the remnant of a supernova, can be used to determine its age and distance.

As the angular velocity of knots within the nebulas filaments is very low it is essential to increase the distances measured and therefore decreasing the effect of measurement error by using images with a significant time span in between them (27 years seemed to be sufficient) and choosing knots on the periphery of the nebula. Additionally it is necessary to do a very precise measurement as the measured changes of the knots are very small and lay between 1,8 and 3,4 mm.

Despite the careful measurements the calculated date of the expansion differs by more than 200 years from 1054 the accepted age of the supernova.

Possible reasons for this irregularity

1) The nebula is not the remnant of the supernova
2) Some of the knots measured in 1973 are not identical to those in the year 2000 image
3) The angular velocity of the knots is not constant

Reason 1 and 2 do not seem very probable as currently it is an accepted fact (page 6 universe 10^{th} edition) that M1 is the wreckage of the supernova of 1054 and even if 1 or 2 knots would have been wrongly identified it would not explain the deviation of more than 200 years!

This leads to the conclusion that the knots have been accelerating outwards supporting the thesis that the angular velocity of the knots is not constant.

The measurements of the other students could be used to verify the conclusion that the knots have been accelerating outwards. If their calculated age also is significant latter than 1054 the conclusion would be strengthened.

We only can measure the maximum linear velocity along the line of sight (maximum shift between red and blue shifted branches).We do not know the 3D shape of the nebula , therefore our best estimate of the distance of the crab nebula is 2032pc using the average angular velocity for the calculation. For sure we only know that the distance lies between 1386pc and 2771pc depending on the shape of the nebula. Further study of linear velocities of additional knots would be very useful in determine the three dimensional shape of the nebula.

Bibliography

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(4) Gingerich, O.November 1977. "Laboratory Excercises in Astronomy – The Crab Nebula." Sky and Telescope. 378-82